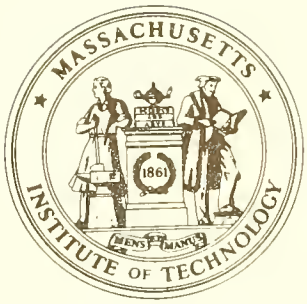

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A Model of the Distribution of Wealth

by

A.B. Atkinson, University of Essex and M.I.T.

Number 123

January 1974

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A Model of the Distribution of Wealth[1]

A.B. Atkinson, University of Essex and M.I.T.

INTRODUCTION

A variety of models have been put forward to explain the distribution of wealth among persons. These models have incorporated a number of factors - such as inheritance, differential savings propensities, life-cycle savings and random shocks - which clearly have a major influence on the degree of inequality. At the same time, they have tended to focus on one particular factor to the partial, or total, exclusion of other important determinants of the distribution. While the stochastic theories, represented by the work of Sargan [16] and Wold and Whittle [21],[2] capture an important aspect of process, they incorporate the minimum of behavioral assumptions concerning the accumulation and transmission of wealth. In contrast to this, the life-cycle savings theories, treated in a general equilibrium context by Meade [13], Diamond [4] and others, have a fully developed explanation for savings behaviour, but the only inequalities allowed for are those between generations (everyone of the same age is assumed to have the same wealth). Finally, those models which do incorporate differences in inherited wealth, notably the work of Meade [12] and Stiglitz [20], make us provision for the other causes of inequality.

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1 An earlier version of this model was presented in a paper to the Berlin Symposium on Planning, August 1973. The present form owes a great deal to constructive comments made by participants in the Symposium and at seminars during my visit to the United States in the Autumn of 1973. I would also like to thank M.A. King, N.H. Stern and J.E. Stiglitz for valuable criticism.

2 See also Steindl [19], Shornocks [18] and, in the case of income distributions, Champernowne [3].

The aim of this paper is to combine in a single model three major sources of inequality which have commonly been treated in isolation: a random process generating new wealth, life-cycle savings, and the transmission of wealth via inheritance.[1] This task is an ambitious one, and it will be clear that this unified treatment has only been achieved at the expense of considerable simplification and that certain elements are in need of further development. The first part of the paper describes the underlying assumptions. The second section examines the behaviour of the model, focussing on the steady-state distribution of wealth and the way in which this depends on the relative importance of the different sources of inequality. The third part shows how the model may be used to analyse the long-run incidence of capital taxation.

---

[1] I should acknowledge at the outset my indebtedness to earlier authors; most of the building blocks for the model are already present in the literature and the only novelty is in the method of assembly.

1. THE MODEL

The first assumptions concern the demographic constitution of the population, and these are designed to be the simplest possible. Individuals live two (equal length) periods and there is no uncertainty about the date of death. Each person has (1+n) children, who are born at the beginning of the second period.[1]

At the beginning of period t, the population entering the first period of its life is given by

$$L_t = L_o (1+n)^t$$

This population consists of the following groups:

(a) Workers who are in the labour force for the whole of the first period of their lives and receive a uniform wage w_k at the end of the period. They consume c_1^t at the end of the first period, leaving savings of $w_t - c_1^t$. In the second period they are retired and at the end of their lives they consume their savings plus the accumulated interest (they leave no bequests): $c_2^{t+1} = (w_t - c_1^t)(1 + r_{t+1})$ where r_{t+1} denotes the rate of return in period t+1. The consumption behaviour is determined by maximising identical utility functions of the form:

$$U(c_1, c_2) = \frac{c_1^{1-\varepsilon}}{1-\varepsilon} + \frac{c_2^{1-\varepsilon}}{1-\varepsilon} \qquad (1)$$

---

[1] These assumptions and those about life-sysle savings draw heavily on the work of Diamond [4].

(where $\varepsilon \geq 1/2$)[1] so that savings at the beginning of period 2 are given by[2]

$$w_t - c_1^t = z(r_{t+1})\, w_t \qquad (2)$$

(b) Entrepreneurs who invest a positive fraction $(1-\alpha)$ of their working lives in risky "entrepreneurial activity",[1] which has a probability r of success and a probability (1-p) of a zero return (where $r < n$). Those who fail as entrepreneurs are left with $w_t\alpha$, and plan their retirement savings to maximise (1), giving savings equal to $\alpha z\,(r_{t+1})\, w_t$. Successful entrepreneurs leave the labour force and acquire a "self-made fortune" by the end of the first period of their lives equal to

$$A_o = m\, x_t \qquad (3)$$

The variable m denotes entrepreneurial 'ability', and a proportion g(m) of the potential working population (defined below) have ability m, and x is the return to successful entrepreneurial ability. The decision to take part in entrepreneurship, which is an indivisible activity, is based on expected utility maximisation,[3] so that those members of the potential working population with ability m will invest according as[4]

---

1 This assumption about the form of the utility function is made solely for convenience and may readily be relaxed.

2 This term is clearly used rather loosely, and I am very conscious of the fact that this does not capture a number of important aspects of the entrepreneurial process.

3 This formulation owes a great deal to Friedman [7].

4 It should be noted that the individual utility function (1) is proportioned to $\frac{W^{1-\varepsilon}}{1-\varepsilon}$ where W denotes the payment received at the end of the first period.

(5)

$$r\frac{(mx_t)^{1-\varepsilon}}{1-\varepsilon} + (1-r)\frac{[w_t\alpha]^{1-\varepsilon}}{1-\varepsilon} \gtrless \frac{w_t^{1-\varepsilon}}{1-\varepsilon} \qquad (4)$$

There will be a value of mx/w ($\equiv\gamma_1$) such that this holds with equality, and for $m \geq m^{\downarrow} \equiv (\alpha_1 w/x)$, people take part in entrepreneurship.[1] The number of entrepreneurs per head of the potential working population is therefore $\int_{m^*}^{\alpha} g\, dm \quad h(m^*)$.

(c) Capitalists who are successful entrepreneurs and their heirs. The effect of successful entrepreneurship is to lead to a change in tastes for accumulation, and the founder of a self-made fortune allocates A_o so as to maximise a utility function which now incorporates a bequest motive

$$U(c_1, c_2, B) = (1-\beta)^{\varepsilon}\frac{c_1^{1-\varepsilon}}{1-\varepsilon} + \frac{c_2^{1-\varepsilon}}{1-\varepsilon} + \frac{B^{1-\varepsilon}}{1-\varepsilon} \qquad (1^1)$$

The bequest B is set aside at the end of the first period of the donor's life.[2] The amount saved at the end of the first period is therefore

$$S_c A_o = [z(r) + \beta(1-z(r)] A_o \qquad (2^1)$$

---

1 In (4) it is assumed that the probability of success (r) is independent of entrepreneurial ability (a person with high m 'thinks big' but does not stand any higher chance of success), but this may be relaxed.

2 It may be checked that, if the entrepreneur forsees the change in tastes, (1^1) is consistent with the valuation placed on success in the first term on the left hand side of (4).

The pattern of inheritance is assumed to be one of primogeniture,[1] so that the entire bequest passes to the eldest son.[2] The son inherits at the end of the first period of his life the bequest set aside by his father one period earlier plus the accumulated interest

$$A_1^{t+1} = B(1 + r_{t+1}) = \beta(1 - z(r_{t+1}))\ A_o^t\ (1 + r_{t+1}) \qquad (5)$$

Eldest sons do not work, and do not form part of the potential working population from which entrepreneurs are drawn, so long as their inheritance exceeds a multiple γ_2 of the wage.[3] They plan their consumption to maximise the utility function (1^1), so that the inheritance received by heirs born at t whose wealth originated v generations earlier is given by

$$A_v^t = A_{v-1}^{t-1}\ [\beta(1 - z(r_t))(1 + r_t)] \qquad (6)$$

Inherited wealth will increase from generation to generation where[4]

---

1 The assumption of primogeniture plays an important role in the analysis, but in my view is likely to be more realistic for large wealth-holdings in the case of Britain than the alternative assumption of equal division among all children. The case of equal division, which may be more relevant to the United States, is examined at length by Stiglitz [20], who also has a brief section on primogeniture to which the present paper owes a great deal.

2 It is assumed that there is always at least one male child, so that the complications caused by marriage can be ignored. For a discussion of the effect of marriage on the concentration of wealth, see Blinder [2].

3 Where it is assumed that $\gamma_2 < \gamma_1$, so that $A_o(m^*)$ is above the critical value.

4 In interpreting this condition it is important to bear in mind that the periods are of approximately generation length and that r may be correspondingly large. The condition is, for example, satisfied where $\varepsilon = 1$, $\beta = 2/3$ and $r = 4\ 1/2\%$ per annum (for a period of 25 years length).

$$S_h(r) \equiv \beta(1 + r)(1 - z(r))-1 > 0 \qquad (7)$$

Younger sons inherit no wealth and enter the labor force in the same way as those whose fathers did not possess fortunes.

The model is summarised in Figure 1, where θ denotes the ratio of heirs to the potential working population, so that the latter is given by $(1/(1+\theta))L_o(1+n)^t$. The dashed line divides the capitalist and working classes. There are clearly close parallels between the present model and the Kaldor/Pasinetti model ([9] and [14]). The capitalist class does not work and it accumulates capital for the sake of accumulation; the working class has no inherited wealth and saves only for life-cycle reasons. However, in contrast to the Kaldor/Pasinetti model, there is a clear link between the class division and the distribution among persons. The origins of the capitalist class are explained not by historical accident but in terms of successful entrepreneurs making the transition from the working class, while the younger sons of capitalists make the reverse transition. Moreover, the distribution of wealth within the capitalist class is determined by the pattern of bequests.

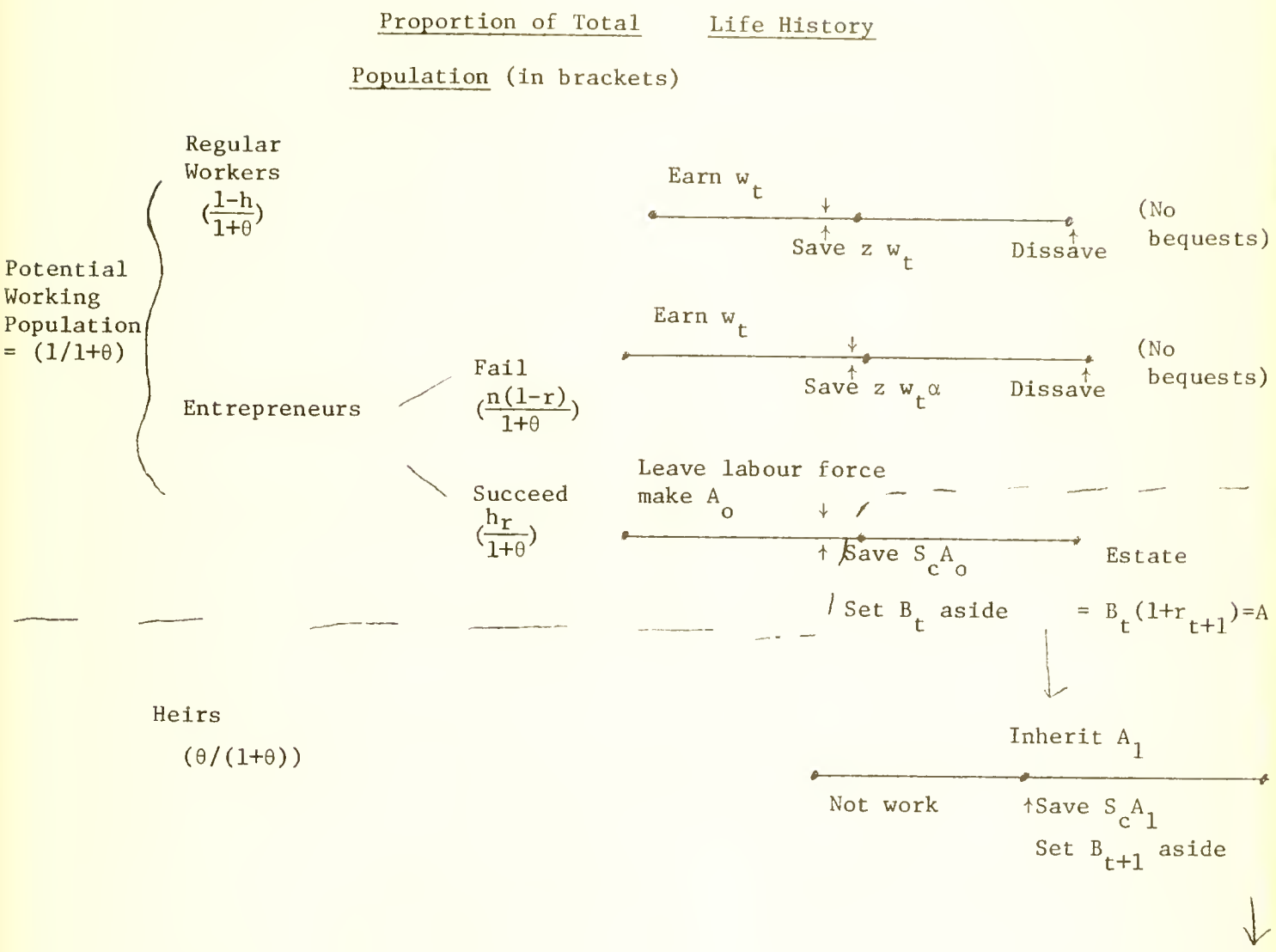
The assumptions about the production side of the economy are chosen for their simplicity. There is a single good which can be used for consumption or investment (there is no depreciation). The unchanging technology gives the following relationship, where y_t denotes output per unit of effective labour and k_t capital per unit of effective labour,

$$y_t = f(k_t)$$

where (8)

$$f^1 > 0,\ f^{11} < 0,\ f^1(0) = \infty,\ f^1(\infty) = 0.$$

Figure 1 Summary of Personal Sector



The total labour supply (in effective units) consists of the regular workers plus α of the time of the unsuccessful entrepreneurs: i.e. $(1 - h) + h\alpha(1 - r)$ per head of the potential working population. Of the total output, a given fraction Π accrues to successful entrepreneurs. This may be seen either as a monopoly rent extracted by virtue of being able to restrict output or as a return to entrepreneurial labour as a factor of production.[1] The total return to entrepreneurs per head of the potential working population is therefore

$$\Gamma x_t \int_{m^*}^{\infty} mg \, dm = \Pi f(k_t)[1 - h + h\alpha(1 - p)] \tag{9}$$

The rate of return and the wage rate are determined by

$$\tau_t = (1 - \Pi) f^1(k_t), \; w_t = (1 - \Pi)(f - k_t f^1) \tag{10}$$

---

[1] In the latter case, the assumption of constant Π implies a particular form of the production function

$$y = \xi^{\Pi} f(k)$$

where ξ denotes entrepreneurs per head of the effective working population. The assumption of constant Π could readily be modified.

2. BEHAVIOUR OF THE MODEL

Attention is focussed on the steady state behaviour of the model, and throughout the remainder of the paper it is assumed that the economy has been in permanent steady state growth. The aggregate behaviour is determined by the supply of capital and the supply of entrepreneurship.

Capital The supply of capital in period t consists of two components. Firstly, there is the life-cycle saving of the retired working class, which is $z(r)w$ per effective worker in period t-1. Since in steady state each group in the population grows at rate $(1+n)$, life-cycle savings amount to $z(r)w/(1+n)$ per head of the current effective working population. Secondly, there is the accumulated wealth of the capitalist class. If we denote by a_t per head of the effective working population the amount inherited at the end of the previous period, then, if (7) is satisfied (see below),

$$(1+n)a = (1 + S_h(r))a + \Pi f(k) \tag{11}$$

where the first term on the right hand side represents accumulated inherited wealth and the second represents the new wealth. Of this capitalist wealth, a fraction $S_c a$ is available to the capital market, so that total capital is

$$k = \frac{z(r)w}{1+n} + \frac{S_c(r)\Pi f(k)}{(n - S_h(r))} \tag{12}$$

It will later be convenient to define $\mu \equiv S_c a/k$, representing the importance of capitalist wealth in the total capital stock.

Entrepreneurship For any given value of k (and hence w) the return to entrepreneurial ability (x) is given by equation (9). Since m^* is a

function of x and h an increasing function, it is clear that there is a unique solution corresponding to any value of k. This then determines h and hence the size of the effective labour force (in relation to the potential labour force).

The steady state behaviour (where (7) is satisfied) depends then on the existence of a solution to (12) where r and w are determined by (10). The conditions for existence and uniqueness of such a solution are discussed in Appendix I. Figure 2 illustrates the special case where $\varepsilon = 1$ (and hence S_c and x constant) and where the production function is Cobb-Douglas. In this case the steady state is unique where condition (7) is not satisfied, inherited wealth will decline from generation to generation and ultimately the eldest son will re-enter the labour force. Discussion of this case is confined to Appendix II.

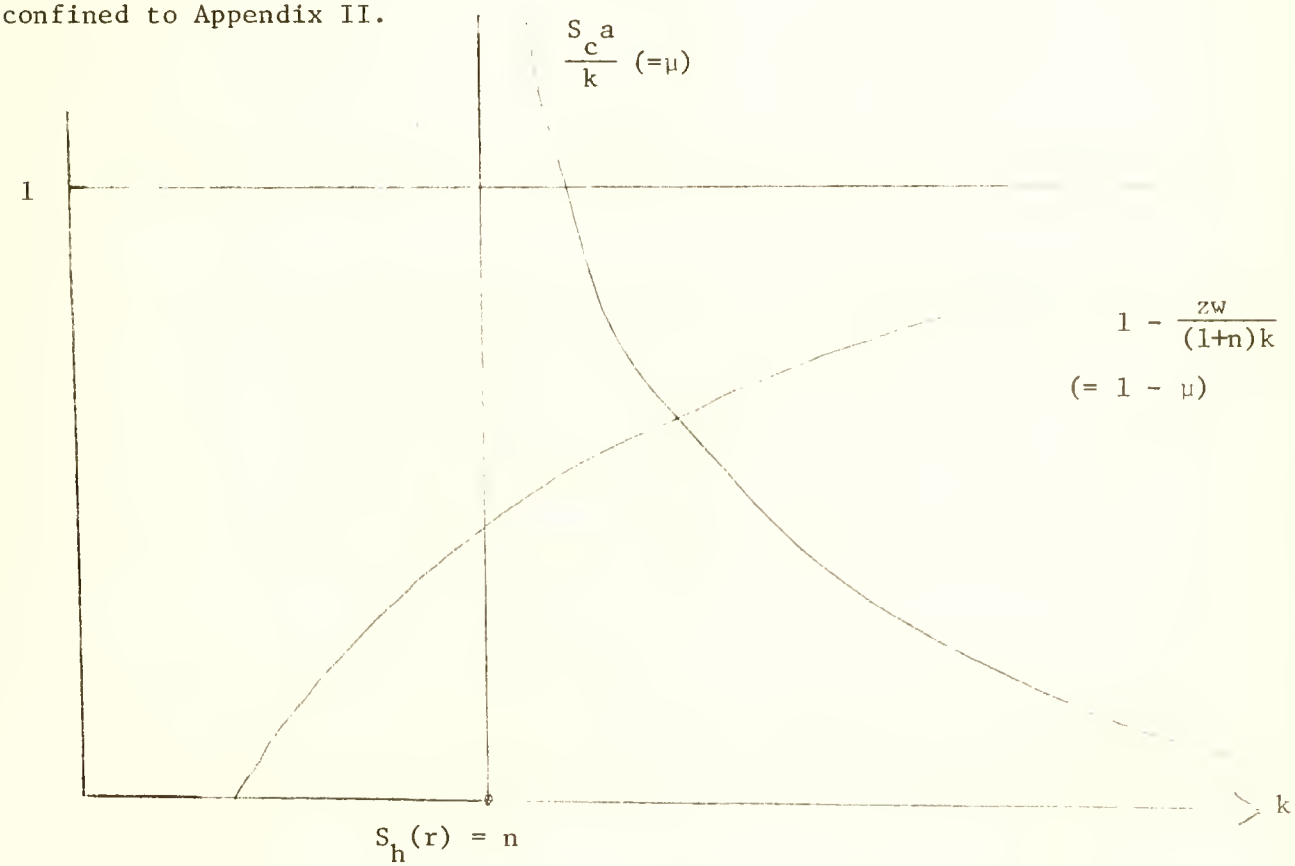


Figure 2 Steady State Solution($\varepsilon = 1$ and Cobb-Douglas production function)

The steady state distribution of wealth may be characterised as follows (under the assumption that (7) is satisfied). At any time t there are $\frac{hp}{1+\theta} L_o(1+n)^t$ successful entrepreneurs, and there are $\frac{hp}{1+\theta} L_o(1+n)^{t-v}$ capitalists whose fortunes originated v periods before. The amount that the latter inherited is given by

$$A_v = A_o[1 + S_h(r)]^{v-1} \qquad v = 1, \ldots.$$

where v = 1 applies to the retired entrepreneur, and v = 2, ... to the successive generations of heirs. If we consider people with the same value of m (and hence A_o in steady state), the number with inherited wealth A_v or more is

$$\frac{\Gamma g(m)}{1+\theta} L_o \sum_{u=v}^{\infty} (1+n)^{t-u} = \frac{\Gamma g(m)}{(1+\theta)n} L_o(1+n)^{t-(v-1)}$$

which may be written as

$$\frac{\Gamma g(\frac{A_o}{x})}{(1+\theta)n} L_o(1+n)^t \, (A_v/A_o)^{-\frac{\log(1+n)}{\log(1+S_h)}} \quad \text{for } A_v \geq A_o \qquad (13)$$

This means that if the ability distribution were concentrated at a single point, the distribution of inherited wealth would be Pareto in form,[1] with an exponent $1/\rho_1 = \log(1+n)/\log(1+S_h)$. Moreover, it can be seen from (13) that the cumulative distribution expressed as a proportion of the total population is independent of t, so that this is a steady state solution.

---

[1] This result is similar to that given by Stiglitz [20], Section 8, but differs in that it is assumed that capitalists have no wage income. This leads to a type I Pareto distribution rather than the type II which can be obtained from the model of Stiglitz. The Stiglitz model does not allow for the generation of new fortunes or for life-cycle savings.

In analysing the general case it is convenient to approximate the distribution by assuming that (13) holds continuously over the range $A_v \geq A_o$. The proportion of the population in a cohort with inherited wealth A or more is then given by

$$1 - \phi(A) \equiv \frac{\Gamma}{n(1+\theta)} \left[h(A/x) + A^{-1/\rho_1} \int_{\underline{A}}^{A} g(A_o/x) \frac{A_o^{1/\rho_1}}{x} dA_o\right] \qquad (14)$$

(where $\underline{A} = m^*x$).[1] Since $(1 - \phi(\underline{A})) = \theta/(1+\theta)$, it follows that $\theta = h(m^*)\Gamma/n$. In (14) the first term represents those whose initial fortunes exceeded A, and the second term corresponds to those who have accumulated sufficient to raise their wealth to A. One case of particular interest is that where g is Pareto, since we are only concerned with the upper tail and a number of distributions have an upper tail which is approximately Pareto in form.[2] Where

$$h\left(\frac{A_o}{x}\right) = h(m^*)(A_o/\underline{A})^{-1/\rho_2} \qquad \text{for } A_o \geq \underline{A}$$

then

$$1 - \phi(A) = \frac{\theta}{(1+\theta)} \left[\frac{\rho_1 (A/\underline{A})^{-1/\rho_1} - \rho_2 (A/\underline{A})^{-1/\rho_2}}{\rho_1 - \rho_2}\right] \qquad \text{for } A \geq \underline{A} \qquad (15)$$

(for $\rho_1 = \rho_2$). The parameter ρ_2 represents the inequality due to self-made fortunes and ρ_1 may be interpreted as corresponding to that caused by inheritance.

---

1 The notation $h(m) = \int_m^\infty g(m)dm$ is used.

2 For a discussion of distributions with this "weak" Pareto property, see Mandelbrot [11].

The analysis to this point has been concerned with the distribution of inherited wealth for a given cohort. The distribution of current wealth-holdings, however, depends on life-cycle savings and on the timing of wealth transfers. If we assume that inherited wealth passes to the heir at the death of the donor (although it is set aside one period earlier), then all wealth is held by people in the second period of their lives. The capitalist class hold $S_c A_v$ in their second period and the retired working population hold zw (or αzw in the case of failed entrepreneurs). The current distribution of wealth among the whole population is therefore as shown in Figure 3.[1]

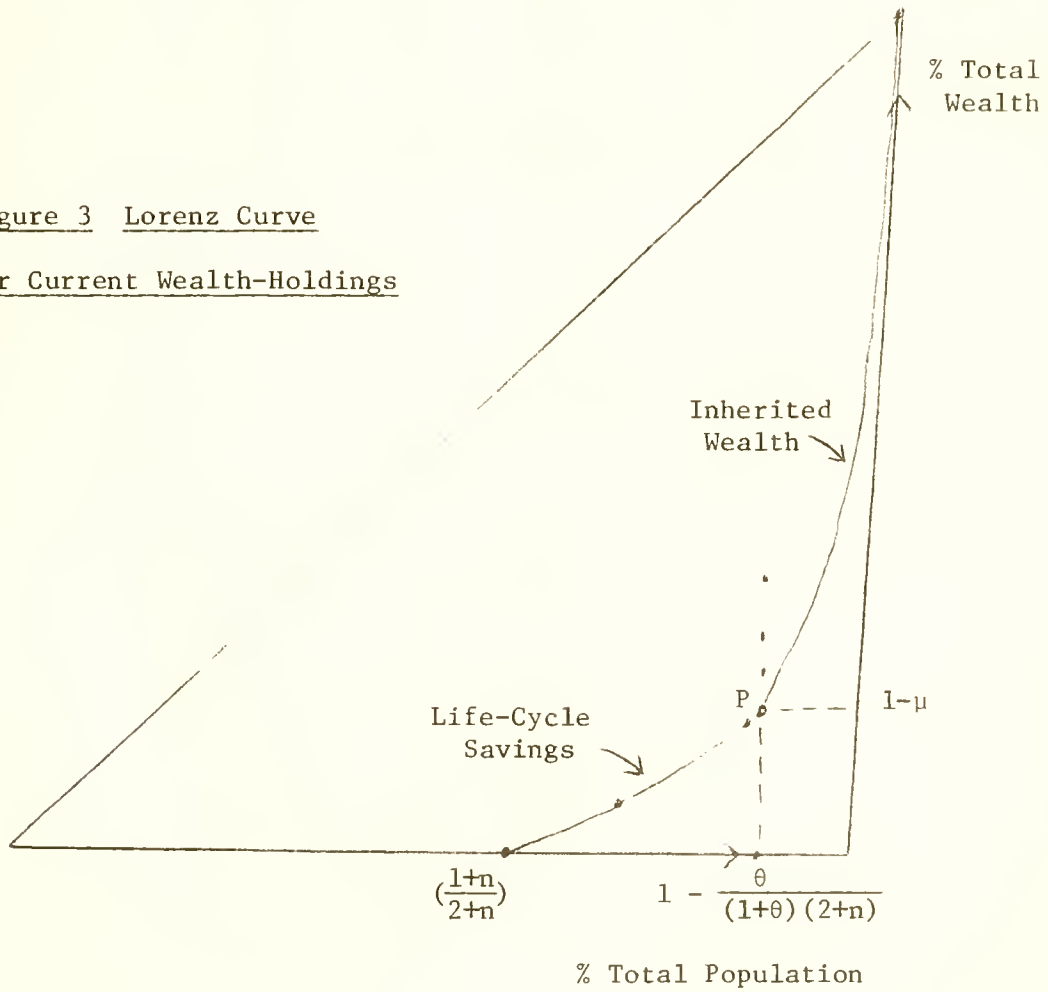


Figure 3 Lorenz Curve for Current Wealth-Holdings

[1] It may be noted that S_c m*x > zw, so that the least wealthy capitalist has larger capital than the representative worker.

Inequality in the current distribution of wealth depends on the relative importance of capitalist wealth μ, the proportion of the population in the capitalist class ($\theta = h(m^*)r/n$), and the distribution of inherited wealth. The first two factors determine the general position of the Lorenz curve; the distribution of inherited wealth determines the stage of the upper tail. In the case where $g(m)$ is approximately Pareto in form over the relevant range ($m \gtrsim m^*$), then the upper tail has the shape given by (15). For large A this approaches a Pareto distribution with an exponent equal to $\max(\ _1, \ _2)$; for lower values there are relatively smaller frequencies than would be predicted by the Pareto case. Preliminary investigation of the British evidence suggests that (15) may provide a better fit to the observed distribution of wealth than the straightforward Pareto form.

3. THE INCIDENCE OF CAPITAL TAXATION

In this section we use the model set out in the earlier part of the paper to examine the long-run incidence of a tax on the income from capital. Initially we consider the balanced budget incidence of taxation as defined by Musgrave, where the tax revenue is used to finance current government expenditure (which enters individual utility functions separably and hence has no effect on saving decisions). There is no government saving.

For later purposes it is convenient to frame the analysis in terms of taxing each of the three sources of income - interest, wages, and entrepreneurial income - at rates τ_c, τ_w and τ_e respectively, so that

$$\tau(1+\tau_c) = (1-\Pi)\ f^1,\ w(1+\tau_w) = (1-\Pi)(f-kf^1) \qquad (16)$$

where the tax is expressed as a fraction of the net return (the equivalent gross rate of tax would be $\tau/(1+\tau)$), and the return to entrepreneurship is reduced to $\Pi f/(1+\tau_e)$. The condition for aggregate equilibrium now becomes

$$k = \frac{z(r)w}{1+n} + \frac{S_c(r)\ \Pi\ f(k)}{(n-S_h(r))(1+\tau_e)} \qquad (17)$$

The effect of taxation may be seen by differentiation of (16) and (17):

$$\begin{bmatrix} 1-\mu(1-r) & -\frac{k}{r}(\mu E_a + (1-\mu)E_z) & -\frac{z}{1+n} \\ (1-\Pi)(-f^{11}) & 1+\tau_c & 0 \\ -(1-\Pi)(-kf^{11}) & 0 & 1+\tau_w \end{bmatrix} \begin{bmatrix} dk \\ dr \\ dw \end{bmatrix} = \begin{bmatrix} -\frac{S_c a}{1+\tau_e}\, d\tau_e \\ -r d\tau_c \\ -w d\tau_w \end{bmatrix} \qquad (18)$$

where the following notation has been introduced:

$$E_a = \frac{r}{(S_c a)} \frac{\partial (S_c a)}{\partial r} \quad , \quad E_z = \frac{r}{z} \frac{\partial z}{\partial r}$$

$$\delta = \frac{f - kf^1}{f}$$

If we also denote by σ the elasticity of substitution between capital and labour

$$\sigma = \frac{f^1(f-hf^1)}{(-kff^{11})}$$

we obtain the following results:

coefficient of	$\frac{d\tau_e}{1+\tau_e}$	$\frac{d\tau_c}{1+\tau_c}$	$\frac{d\tau_w}{1+\tau_w}$
$D \frac{dk}{k} =$	$- \mu$	$- [\mu E_a + (1-\mu) E_z]$	$- (1-\mu)$
$D \frac{dr}{r} =$	$\frac{\mu\delta}{\sigma}$	$- [1-(1-\delta)[\mu + \frac{1-\mu}{\sigma}]]$	$(1-\mu) \frac{\delta}{\sigma}$
$D \frac{dw}{w} =$	$- \mu \frac{(1-\delta)}{\sigma}$	$- \frac{(1-\delta)}{\sigma} [\mu E_a + (1-\mu) E_z]$	$- [(1-\mu(1-\partial) + \frac{\delta}{\sigma} (\mu E_a + (1-\mu) E_z)]$

$D (1+\tau_w)(1+\tau_c)$ is the determinant of the coefficient matrix and is assumed positive (see Appendix I).

(a) Shifting of Capital Taxation

Let us consider first the case of a tax falling solely on interest income ($\tau_e = \tau_w = 0$). The most interesting aspect concerns the effect on the after-tax rate of return, which rises or falls as

$$\sigma \lessgtr \frac{(1- \ )(1- \ )}{1- \ (1- \ )} \qquad (19)$$

In other words, there may be long-run shifting (defined in terms of the rate of return) in excess of 100%. Moreover, this possibility depends crucially on the relative importance of inherited and life-cycle wealth. In the limiting case where $\mu \to 1$, then more than 100% shifting is not possible; on the other hand, where = 0, the condition becomes $\sigma < 1 - \delta$, which may be satisfied for low values of the elasticity of substitution.[1] If the tax were to fall not only on interest income but also on the return to entrepreneurship (a general tax on all non-wage income), then $\tau_e = \tau_c$ means that the degree of shifting is increased (for $\mu > 0$).

The possibility of more than 100% shifting was demonstrated by Diamond [5] in a purely life-cycle model.[2] The advantage of the unified treatment given here of life-cycle and inherited wealth is that it allows us to identify more clearly the factor responsible and to relate the life-cycle results to those from class savings models. If we unite the capital accumulation relationship (with $\tau_e = 0$) as

$$(1+n)k = z(r)w + S_c(r)\Pi f(k) + (1+S_h(r))a \qquad (20)$$

we can see that the difference lies in the importance of saving out of wage income. In the extreme case of Pure Life-Cycle Accumulation (where $\Pi = 0$ and hence $\mu = 0$), then

$$nk = \frac{n}{1+n}\, z(r)\, w \qquad (21)$$

---

1 $(1-\delta)/\delta$ equals the relative gross shares of capital and labour in output, so that $1-\delta = 1/3$ might be a reasonable value. σ would have, therefore, to be rather low.

2 Strictly his result applies to the differential incidence of τ_c and a lump-sum tax on the older generation, rather than to balanced budget incidence.

This can be interpreted as an "inverted" Kaldorian model where only wage-earners save. The possibility or $dr/d\tau_c > 0$ in this case may be seen from Figure 4. The condition $\sigma \gtrless 1 -$ determines whether k/w is an increasing or decreasing function of k. If $\sigma < 1 - \delta$, then the effect of the tax (which shifts the $z/(1+n)$ curve to the dashed position) is for $z(r)$ and hence r to rise ($\partial z/\partial v$ and $z/1+n$ cutting from above is required for $D > 0$). In the other extreme case of Pure Capitalist Accumulation (where $z = 0$ and $\mu = 1$),

$$nk - S_c(r)\Pi f(k) + \left(\frac{S_h(r)}{r}\right) rk \tag{22}$$

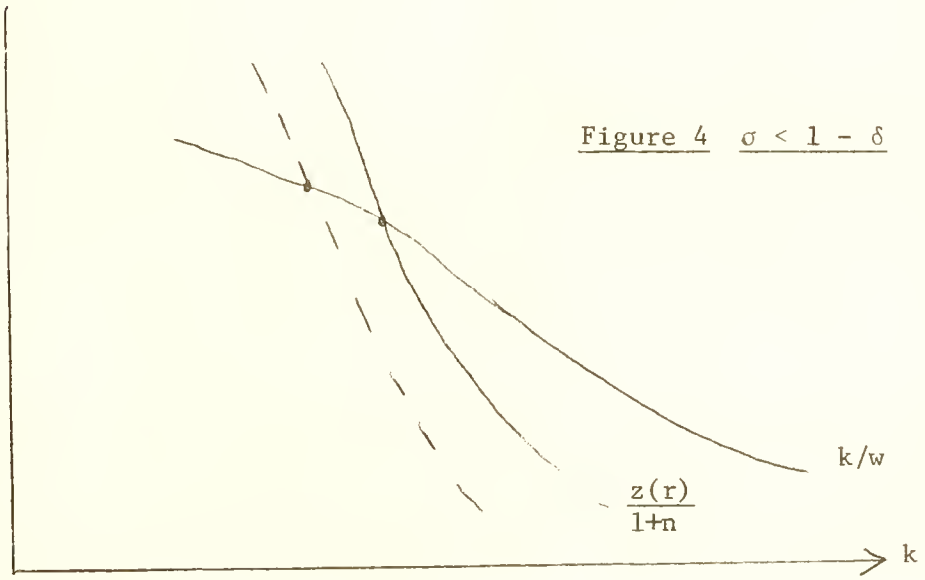


Figure 4 $\sigma < 1 - \delta$

The term w/k no longer appears and the possibility of 100% shifting does not exist.[1] The accumulation relationship (22) may be seen as an extended, two-class capitalist model of the Kaldorian type, where the propensity to save out of entrepreneurial income is higher than that out of the interest on inherited wealth.[2] - This seems a very natural division of the capitalist class into "old" and "new" wealth.

(b) Capital Taxation and the Distribution of Wealth

So far we have considered the impact of taxation only in terms of the aggregate equilibrium.[3] The effect on the current distribution of wealth depends firstly on how μ and m^* are changed by the tax. The effect on μ is given by (focussing on τ_c)

$$D\,\frac{d\mu}{\mu} = -\,(1-\mu)\left[E_a\left(1-\frac{1-\delta}{\sigma}\right) - \delta E_z\right]\frac{d\tau_c}{1+\tau_c} \qquad (23)$$

From this it is clear that μ may rise (e.g. where $\sigma = 1$ and $E_z > E_a$) so that a tax on capital may increase the importance of inherited wealth relative to life-cycle savings. The effect on m^* can be seen from (9) which gives

---

1 These results may be compared with those obtained from purely 'class' savings models. Although Feldstein [6] does not discuss the possibility of more-than-100% shifting, from his equation (18) it is clear that r rises where $(1-\alpha-\sigma)\,S_L$ is negative, where S_L is the propensity to save out of wages (and it is assumed that the denominator is positive). See also Krzyzaniak [10] and Sato [17].

2 Since $S_c = \beta$ and $\frac{S_h}{r} = \beta - \left(\frac{1-\beta}{r}\right)$.

3 It may be noted that the aggregate incidence in terms of factor returns is independent of the effect on the supply of entrepreneurship. This would not carry over if Π varied with m^*.

$$(\frac{\gamma 1}{m^*})(\int_{m^*}^{\infty} mg\ dm)/(1 - h + h\alpha(1-r)) = \frac{\Pi f(k)}{w} \qquad (24)$$

so that $dm^*/d\tau_c$ is of the sign of $(dk/d\tau_c)(1-\sigma)$. So that if $E_a\ E_z > 0$ - which ensures $dk/d\tau_c$ negative - and $\sigma > 1$, then m^* rises and a smaller fraction of the population enters the capitalist class. The point P on the Lorenz curve (see Figure 3) may therefore be moved away from the diagonal as a result of capital taxation: e.g. where $1/2 \leq \varepsilon < 1$ and $\sigma > 1$, it is possible that $E_z > E_a > 0$, so that both μ and m^* rise.

Taxation of capital also affects the current distribution via the degree of concentration of inherited wealth; this depends on ρ_1, which is an increasing function of τ. In the case where the upper tail of the distribution of abilities is Pareto in form, the Lorenz curve is given by

$$1 - \phi(A) = \frac{\theta}{1+\theta}\left[\frac{\rho_1 (A/\underline{A})^{-1/\rho_1} - \rho_2 (A/\underline{A})^{-1/\rho_2}}{\rho_1 - \rho_2}\right] \qquad (25)$$

and

$$1 - \Omega(A) = \mu\left[\frac{\rho_1 (1-\rho_2)(A/\underline{A})^{-1/\rho_1+1} - \rho_2 (1-\rho_1)(A/\underline{A})^{-1/\rho_2+1}}{\rho_1 - \rho_2}\right] \qquad (26)$$

where $A \geq \underline{A}$ and $\rho_1 \neq \rho_2$. $(1-\Omega(A))$ denotes the cumulative share in current wealth of those with inheritances of A or more. It can be shown that for given θ, μ (i.e. fixed P in Figure 3), the effect of a rise in ρ_1 is to increase inequality in the distribution of wealth within the capitalist class. More than 100% shifting of capital taxation leads therefore to greater inequality within the capitalist class.

(c) Differential Incidence and Lifetime Utility

The fact that the distribution of wealth may become more concentrated as a result of capital taxation does not imply that the distribution of lifetime utilities is necessarily more unequal. Let us consider the welfare of a representative worker, which may be written in terms of his indirect utility function[1]

$$V(w,r) \qquad \text{where} \qquad \frac{\partial V}{\partial w} = c_1^{-\varepsilon} \qquad \frac{\partial V}{\partial r} = (c_1^{-\varepsilon})\ \frac{z(r)w}{1+r}$$

Moreover, to avoid the problems raised by the valuation of public expenditure, let us now consider a purely redistributive tax where the revenue from taxing capital income (τ_c) is used to finance a subsidy on wages ($-\tau_w$) - a differential incidence question. In other words, total revenue per effective worker

$$\tau = \tau_w w + \tau_c\ rk = 0 \tag{27}$$

The total effect of a change in τ_c is found by differentiating (16), (17) and (27)

---

[1] The indirect utility function for the failed entrepreneur is proportional to V, so that in this sense the interests of the working class are identical.

$$\begin{bmatrix} 1-\mu(1-\delta) & -\frac{k}{r}(\mu E_a+(1-\mu)E_z & -\frac{z}{1+n} & 0 \\ (1-\Pi)(-f^{11}) & 1+\tau_c & 0 & 0 \\ (1-\Pi)(kf^{11}) & 0 & 1+\tau_c & w \\ \tau\tau_c & k\tau_c & \tau_w & w \end{bmatrix} \begin{bmatrix} dk/d\tau_c \\ dm/d\tau_c \\ dw/d\tau_c \\ d\tau_w/d\tau_c \end{bmatrix} = \begin{bmatrix} 0 \\ -r \\ 0 \\ -rk \end{bmatrix} \tag{28}$$

which gives (where $w\Delta(1+\tau_c)$ is the determinant of the coefficients and is

which gives (where $w\Delta(1+\tau_c)$ is the determinant of the coefficients and is

again positive by assumption - see Appendix I)

$$(1+\tau_c)\frac{\Delta}{r}\frac{dr}{d\tau_c} = \frac{rz_{\tau c}}{1+n} - (1-\mu(1-\delta)) \tag{29}$$

$$(1+\tau_c)\frac{\Delta}{w}\frac{dw}{d\tau_c} = \frac{rk}{w}[(1-\mu(1-\delta)) - \tau_c(\mu E_a + (1-\mu)E_z)] \tag{30}$$

so that the change in life-time utility is proportional to

$$[1-\mu(1-\delta)][1-(1-\mu)\frac{1+n}{1+r}] + \tau_c[(1-\mu)\frac{rz}{1+\tau} - \mu E_a - (1-\mu)E_z] \tag{31}$$

Evaluating at $\tau_c = 0$, it is clear that an infinitesimal tax on capital income used to subsidise wages will raise life-time utility for the representative worker where

$$1 + r > (1+n)(1-\mu) \tag{32}$$

a sufficient condition being that the rate of interest exceed the rate of growth.[1] It is interesting to contrast this result with that of Hamada [8],

---

[1] This result may seem counter-intuitive, since it indicates that a tax on capital is desirable when capital is below the Golden Rule level; it has, however, to be remembered that the condition $r \gtrless n$ plays two roles in this model and that it also indicates that the allocation of consumption to the older generation is greater than the biological optimum (see Diamond [4], p. 1129).

who argued that in a comparison of steady-state parts "no transfer is the best policy for the workers", so that a tax on capital used to subsidise wages would make the working population worse off. The reason for Hamada's striking result - and its dependence on the very special model which he considers - may be seen if we consider the limiting case $\mu = 1$. The workers are then only concerned with the net wage (since they do no saving), and (30) has the sign of[1]

$$\delta - \left(\frac{r\beta}{n-S_h}\right) \tau_c$$

The model used by Hamada, however, was of the Pasinetti type and the condition for steady-state equilibrium considered by him was $n = S_h$. In the more general model examined here, this condition is not satisfied in the steady-state and Hamada's conclusion ceases to be valid.

---

[1] Where $z = 0$ is substituted in the expressions for S_c and S_h.

4. CONCLUDING COMMENTS

The analysis of this paper could be developed in a variety of ways. It would be possible to examine in greater detail the impact of taxation, extending the discussion to include taxes on the transfer of wealth, the impact of income taxation on risk-taking, the implications of government capital formation or government debt, and the benefit to the working class from abolishing inheritance. These concluding comments are focussed, however, on the specification of the underlying model.

One of the most important features of the model discussed in this paper is that the steady-state solution is characterised by permanent inequality in the distribution of wealth. This may be contrasted with the earlier deterministic models, such as that of Meade [13] where inequality within generations is ruled out by assumption, and that of Stiglitz [20] where the stable steady-state solution considered in the main part of the paper has an asymptotically equal distribution of wealth. There are three main factors responsible for the difference:

(i) differential tastes for accumulation on the part of the working and capitalist classes,

(ii) the pattern of inheritance - primogenitive rather than equal division,

(iii) unequal returns to entrepreneurship.

None of these factors is by itself essential to the analysis. The class savings behaviour could be replaced by the assumption of identical utility functions for workers and capitalists. The assumption about the pattern of bequests, which is clearly at the opposite extreme from that

of equal division, could be replaced by an intermediate case likely to be closer to reality.[1] The unequal returns to entrepreneurship may be given a variety of alternative interpretations, including differing degrees of risk aversion or optimism. However, the results would not survive the removal of all these sources of inequality. There is a celebrated exchange in which F. Scott Fitzgerald is reported to have said "You know, Ernest, the rich are different from us" and Hemingway to have replied "Yes, I know. They have more money than we do". Although Fitzgerald felt that he came out the worse from this exchange[2], he had an important point. In explaining inequality in the distribution of wealth, there must be some way in which the rich - or the founders of their fortunes - differ from the rest of the population.

---

1 A form which captures the asymmetric division often observed in practice is where all heirs receive an equal share of the estate up to some "comfortable provision", and any estate in excess of this amount goes to a single heir. This intermediate case is rather different from that assumed by Blinder [2], who assumes that estates are always divided in the same (not necessarily equal) proportions.

2 In fact the exchange never took place according to C.C. Kirstein (The Rich - Are They Different?). Heningway referred to it, without claiming that he himself made the retort, in the first publication of "The Snows of Kilimanjaro" but Fitzgerald's name was dropped when it appeared in book form.

Appendix I

The steady-state solution (in the no-tax case) is characterised by (where condition (7) is satisfied)

$$k = \frac{z(r)w}{1+n} + \frac{S_c(r)\Pi f(k)}{n - S_h(r)} \tag{12}$$

where $$\tau = (1-\Pi)\ f^1(k),\ w = (1-\Pi)(f-kf^1) \tag{10}$$

It is convenient to consider first

$$\mu = \frac{S_c(r)\Pi f(k)}{[n-S_h(r)]k}$$

From the definition of $1 + S_h(r) = \beta(1+r)(1-z)$, it can be seen that $\varepsilon \geq 1/2$ ensures that S_h is an increasing function of r. We may therefore define $\hat{k}$ such that $S_h[(1-\Pi)\ f^1(\hat{k})] = n$. It follows that the steady-state solution has $k > \hat{k}$. From the assumptions about the production function (8), both r and (f/k) tend to zero as $k \to \infty$. From the relationship[1]

$$1 - z = (1 + (1+r)^{1/\varepsilon-1})^{-1}$$

it is clear that as $k \to \infty$, than $z \to 1/2$, $S_c \to (1+\beta)/2$ and $S_h \to \beta/2 - 1$ so that $\mu \to 0$ as $k \to \infty$. A sufficient condition for μ to decline monotonically (for $k > \hat{k}$) is that $E_z \geq 0$ (i.e. $\varepsilon \leq 1$).

If we turn to consider

$$1 - \mu^1 \equiv \frac{z(r)w}{(1+r)k}$$

---

[1] Obtained from the conditions for utility maximization for the working class.

the conditions on the production function ensure that $w/k \to 0$ as $k \to \infty$, so that $\mu^1 \to 1$. Moreover, $\mu^1 \leq 1$. This ensures that there exists an equilibrium with $\mu = \mu^1$. Sufficient conditions for μ to rise monotonically in the relevant range are $E_z \gtrsim 0$ and $\sigma \gtrsim (1-\delta)$ for $k > \hat{k}$. These are sufficient for the uniqueness of the steady-state solution (as in the case illustrated in Figure 2 in the text).

In general there may be multiple equilibria, as illustrated in Figure A1. The argument of the previous paragraph only ensures the existence, however, of 'regular' equilibria where the $1-\mu^1$ curve cuts from below (such as A or C). The condition for such an intersection (evaluating at the equilibrium) is that

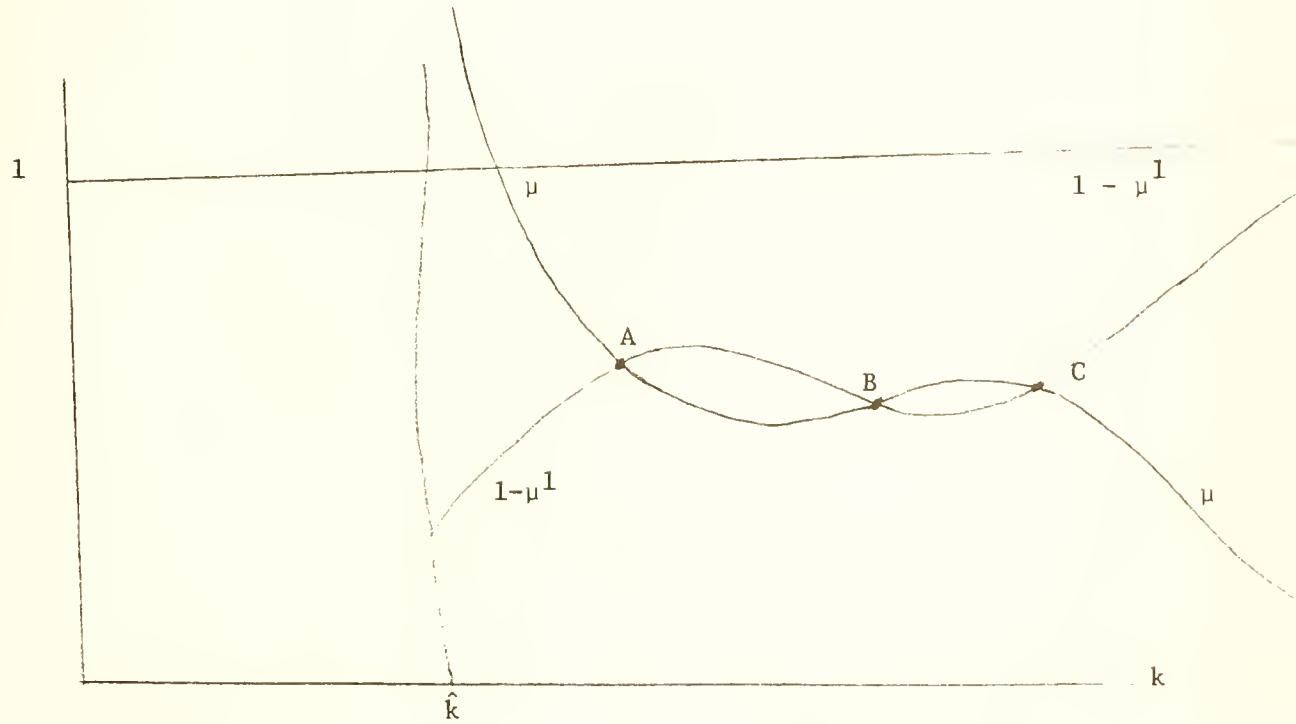


Figure A1

$$\frac{d}{dk}\left[k - \frac{z(r)w}{1+n} - \frac{S_c \Pi f}{n-S_h}\right] > 0$$

which reduces to the condition $D > 0$ where $D(1+\tau_c)(1+\tau_w)$ is the determinant of the coefficient matrix in (18)[1].

In the case of differential incidence, the government is assumed to adjust τ_w to maintain the revenue yield. If τ_w is adjusted to ensure that (25) holds, then the condition for a 'regular' equilibrium becomes $\Delta > 0$, where $\Delta w(1+\tau_c)$ is the determinant of the coefficients of (26).

Appendix II

The analysis of the text was based on the assumption that condition (7) was satisfied at the steady-state equilibrium so that inherited wealth increased from generation to generation. Where this is not satisfied, the eldest son will re-enter the labour force at the point (V*) where A_{V*} falls below γ_2 w. His taxes are then assumed to revert to those of the working class, so that he leaves no bequests[1].

If we consider people of ability m, the total inherited wealth is

$$\frac{\Gamma g(m) \text{ x m } L_t}{(1+\theta)(1+n)} \int_{V=1}^{V*} \left(\frac{1+S_h}{1+n}\right)^{v-1} = \frac{\Gamma g(m) \text{ x m } L_t}{(1+\theta)(n-S_h)} \left[1 - \left(\frac{1+S_h}{1+n}\right)^{V*}\right]$$

where

$$x \text{ m } (1+S_h)^{V*} \leq \gamma_2 w.$$

As in the text, it is a convenient approximation to treat generations in a continuous manner, so that this relationship holds with equality and we can unite the total wealth of people with ability m

$$\frac{\Gamma \text{ x } L_t}{(1+\theta)(n-S_h)} \left[mg - mg \left(\frac{\gamma_2 w}{x m_1}\right)^{1-\rho_1}\right]$$

---

[1] For "shirtsleeves to shirtsleeves in three generations", i = 2.

Comparing this with (9) it is clear that $\underline{a}$ now becomes

$$\frac{\Pi f(k)}{n-S_h}\left[1 - \left(\frac{\gamma_2 w}{x}\right)^{1-\rho_1} \frac{\int_{m^*}^{\infty} m^{\rho_1} g \, dm}{\int_{m^*}^{\infty} m \, g \, dm}\right]$$

where ρ_1 $\left(= \frac{\log(1+S_h)}{\log(1+n)}\right)$ is now negative. The supply of capital is now a function of m*.

The number of people with inherited wealth A or more is given by (again treating generations in a continuous manner)

$$1 - \phi(A) = \frac{\Gamma L_t}{1+\theta} \int_0^{\infty} 1 - G(A_e^{-S_h t}) \, e^{-nt} \, dt$$

In the case where g is Pareto,

$$1 - \phi(A) = \frac{\theta}{1+\theta} \frac{\rho_2}{\rho_2 - \rho_2} (A/\underline{A})^{-1/\rho_2}$$

i.e. the distribution of wealth minors the distribution of entrepreneurial abilities.

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